Lectures on Electromagnetic theory I

PH 2151

Lecture 2
(The three coordinate systems)

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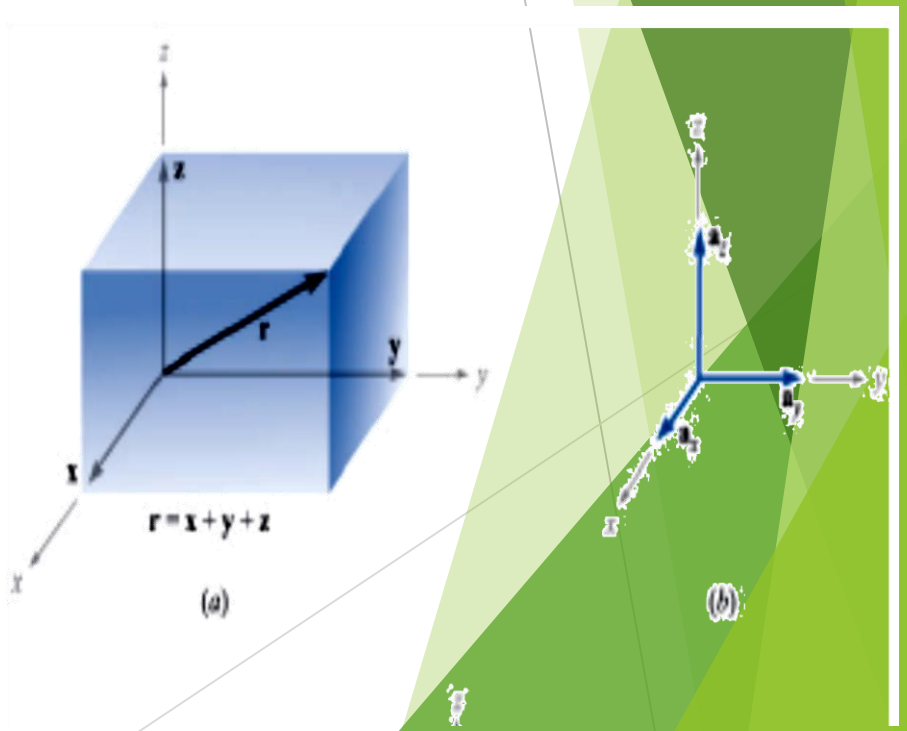
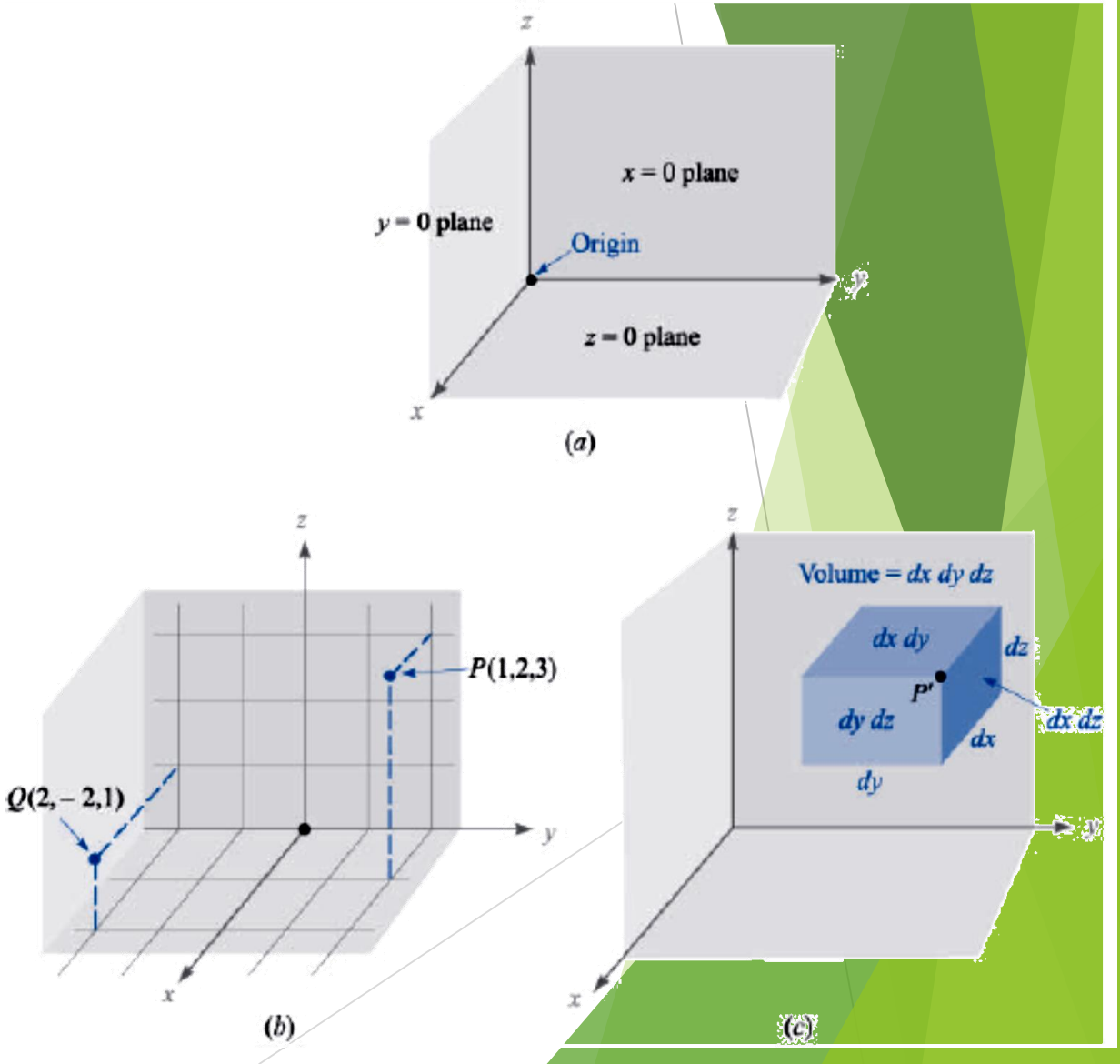
The cartesian coordinate system

- The coordinates are x y z
- The position vector r=x a_x +y a_y +z a_z.
- The perpendicular unit vectors are a_x, a_y, a_z.



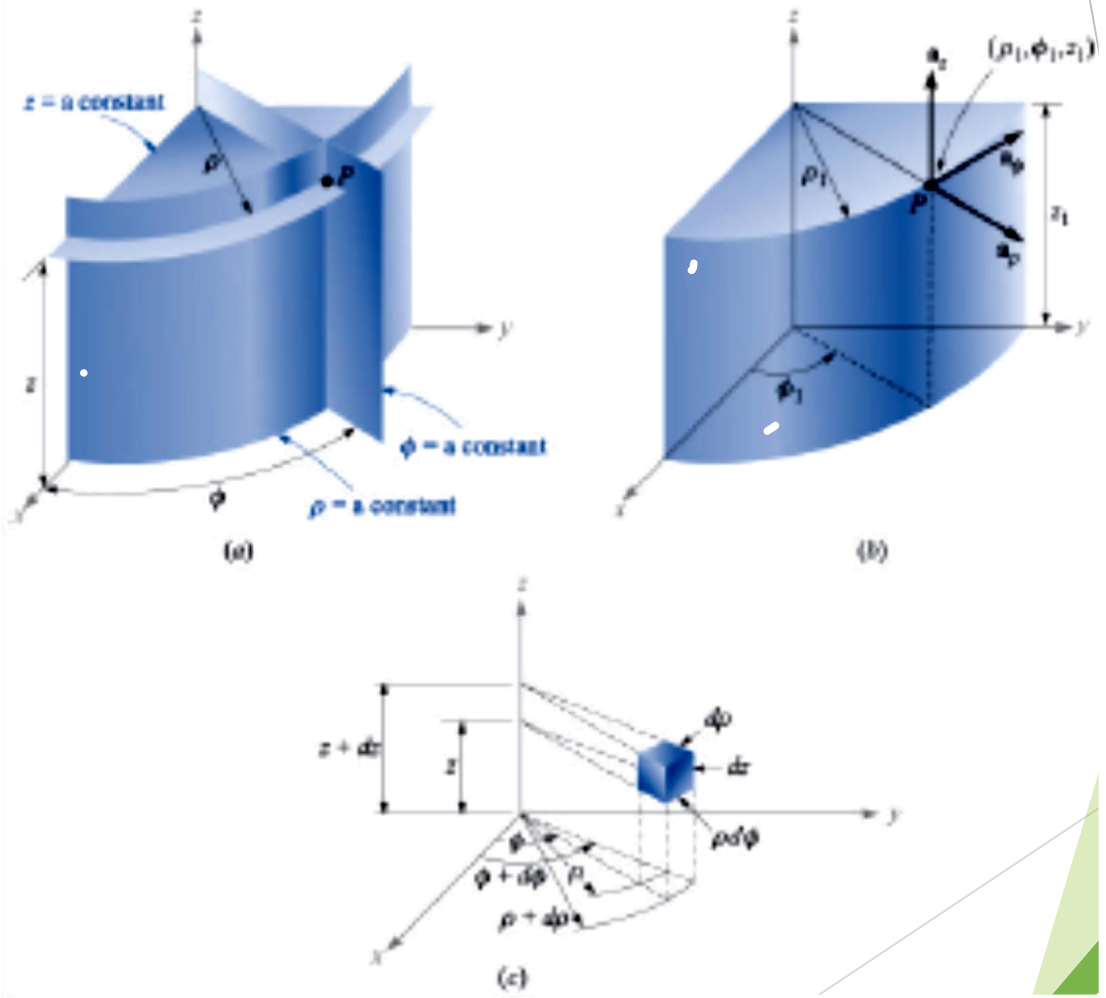
The cartesian coordinate system

- The displacement element
  dl = dx a_x +dy a_y + dz a_z
- The volume element
  dv= dxdydz



Circular cylindrical coordinate system

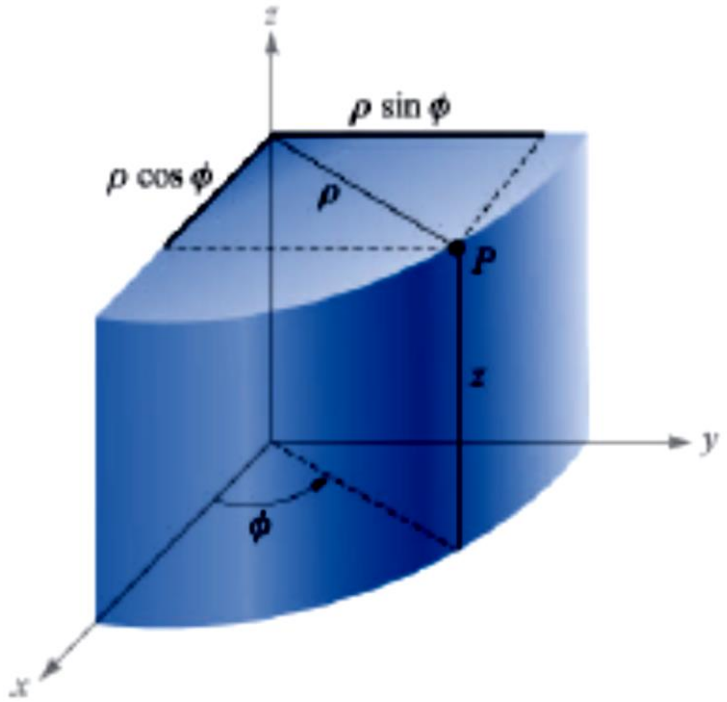
- The coordinates are ρ, φ,z
- The unit vectors are $\boldsymbol{a}_\rho$, $\boldsymbol{a}_\varphi$, $\boldsymbol{a}_z$.
- The displacement element
  dl = dρ $\boldsymbol{a}_\rho$+ρdφ $\boldsymbol{a}_\varphi$ + dz $\boldsymbol{a}_z$
- The volume element
  dv= dρ ρdφ dz



Circular cylindrical coordinate system

- The relation of the variables in rectangular and cylindrical coordinate systems and vise versa is :

$$x = \rho\cos\phi \qquad \rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$
$$y = \rho\sin\phi \qquad \phi = \tan^{-1}\frac{y}{x}$$
$$z = z \qquad z = z$$



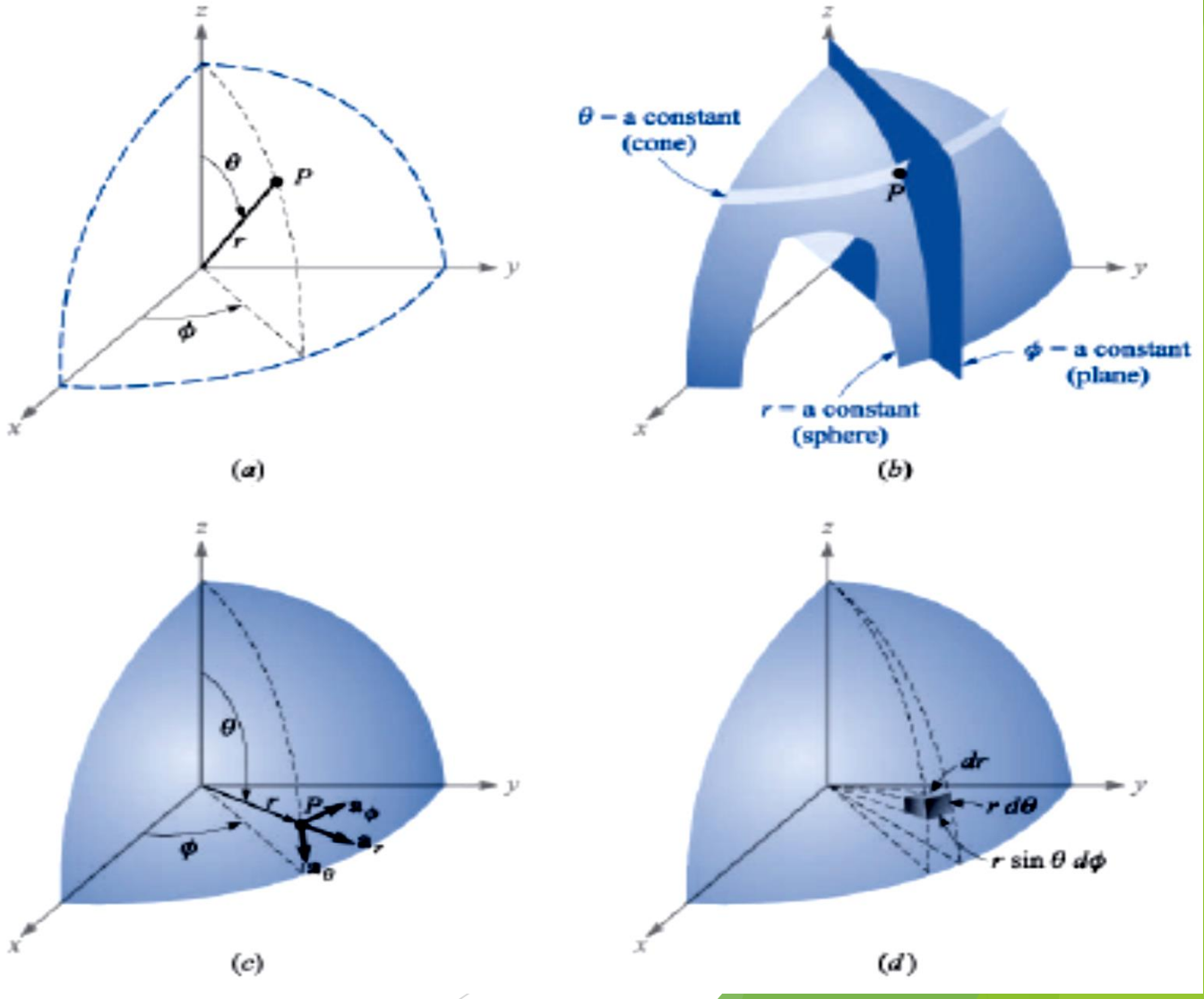
Circular cylindrical coordinate system

Dot products of unit vectors in cylindrical and cartesian coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

The spherical coordinate system

- The three coordinates are r, θ, Φ
- The unit vectors are $\mathbf{a}_r$, $\mathbf{a}_\theta$, $\mathbf{a}_\Phi$.
- The displacement element dl= dr $\mathbf{a}_r$+rdθ $\mathbf{a}_\theta$ + rsinθ $d\Phi$ $\mathbf{a}_\Phi$
- The volume element dv= dr rdθ r sinθ dφ



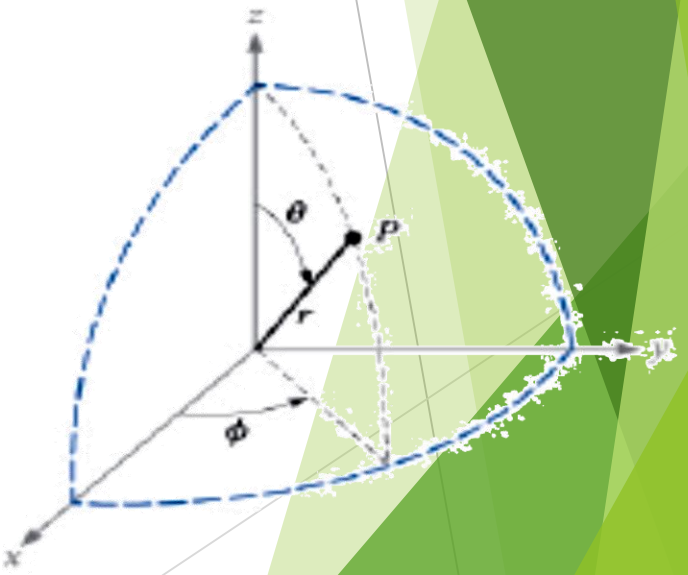
The spherical coordinate system

- The relation of the variables in rectangular and spherical coordinate systems and vise versa is :

$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$

$$y = r\sin\theta\sin\phi \qquad \theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$

$$z = r\cos\theta \qquad \phi = \tan^{-1}\frac{y}{x}$$



The Spherical coordinate system

Dot products of unit vectors in spherical and cartesian coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Problems:

1. Find the vector directed from the point (10,3π/4,π/6) to the point (5,π/4,π).
2. Find the distance between the points (2, 6π, 0) & (1, π , 2).
3. Using the spherical coordinate, obtain the volume between $1 \leq r \leq 2$ m , $o \leq \theta \leq \pi/2$ and $o \leq \varphi \leq \pi/2$.
4. Transform the vector A= $y a_x$ +x a_y + $\frac{x^2}{\sqrt{(x^2+y^2}}$ a_z into cylindrical coordinate system .
5. Use the cylindrical coordinates to find the surface area of cylinder with radius a and height h.